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COMBINING UNIVERSAL EPISTEMOLOGY WITH FORMAL AXIOLOGY IN A MULTIMODAL FORMAL AXIOMATIC THEORY "SIGMA + 2C", AND PHILOSOPHICAL FOUNDATIONS OF MATHEMATICS

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Abstract. The paper is devoted to investigating Kant's apriorism underlying Hilbert's formalism in philosophical foundations of mathematics. The target is constructing a formal axiomatic theory of knowledge in which it is possible to invent formal inferences of formulae-modeling-Hilbert-formalism from the assumption of Kant apriorism concerning mathematics. The s*cientific novelty*: a logically-formalized axiomatic system of universal philosophical epistemology called "Sigma +2C" is invented for the first time as a generalization of the already published formal epistemology system "Sigma +C". In comparison with "Sigma +C", a new symbol is included into the object-language-alphabet of Σ+2C, namely, the symbol standing for the *perfection*-modality "it is *complete* that…". Also, one of axiom-schemes of "Sigma +C" is generalized in "Sigma + 2C". In "Sigma +2C", it is proved deductively that under the assumption of *a-piori*-ness of mathematical knowledge, its completeness and consistency are equivalent.

Keywords. formal axiomatic theory of knowledge; a-priori knowledge; empirical knowledge; Kant's apriorism; Hilbert's formalism; Gödel's incompleteness theorem; two-valued algebraic system of formal axiology.

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ОБЪЕДИНЕНИЕ УНИВЕРСАЛЬНОЙ ЭПИСТЕМОЛОГИИ С ФОРМАЛЬНОЙ АКСИОЛОГИЕЙ В МУЛЬТИМОДАЛЬНОЙ ФОРМАЛЬНОЙ АКСИОМАТИЧЕСКОЙ ТЕОРИИ «СИГМА + 2С», И ФИЛОСОФСКИЕ ОСНОВАНИЯ МАТЕМАТИКИ

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Аннотация. Статья посвящена исследованию кантовского априоризма, являющегося предпосылкой формализма Гильберта в философских основаниях математики. Цель – построение некой формальной аксиоматической теории знания, в которой возможно построение формальных выводов формул, моделирующих формализм Гильберта, из допущения кантовского априоризма математического знания. *Научная новизна*: впервые построена некая логически формализованная аксиоматическая система универсальной философской эпистемологии «Сигма + 2С» как обобщение уже опубликованной системы формальной эпистемологии «Сигма + С». В сравнении с «Сигма + С», некий новый символ включен в алфавит языка-объекта «Сигма + 2С», а именно, символ, обозначающий модальность *идеала (совершенства):* «это *полно*, что …». Также, в системе «Сигма + 2С», одна из схем аксиом системы «Сигмы + С» существенно обобщена. В «Сигма + 2С» дедуктивно доказано, что при допущении априорности математического знания, его полнота и непротиворечивость эквивалентны.

Ключевые слова: формальная аксиоматическая теория знания; априорное знание; эмпирическое знание; априоризм Канта; формализм Гильберта; теорема Гёделя о неполноте; двузначная алгебраическая система формальной аксиологии.

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1. *Introduction*

There are infinitely many different modal logics. The number of possible combinations of different kinds of modalities is immense. Even within the scope of modal logic of knowledge we need a set of *significantly different* modalities called "knowledge"; various combinations of which make different *multimodal* epistemic logics. As in the intellectually respectable definitions of the notion "knowledge", the words "true" and "justified (proved)" or "provable" are exploited necessarily, the modal logic treating *truth as modality* and the modal logic treating *provability (justifiability) as modality* are indispensable for epistemology. In content philosophy the word-homonym "knowledge" is naturally connected with many other modal terms (alethic, deontic, axiological, et al), consequently, while inventing and elaborating a hypothetical multimodal formal axiomatic system of universal philosophical epistemology, one has to utilize not only proper-epistemic modalities but also many other concepts of modal metaphysics. This is just what I am to do in the present article, namely, I am to invent (construct) a novel logically formalized axiomatic system of *multimodal* philosophy of knowledge. However, the concrete theme and the goal of the paper necessitate a restriction of the set of different kinds of modalities to be involved into the discourse.

The list of various kinds of modalities to be taken into an account in this article is determined by the subject-matter and target of the research. At present moment I am equipped with the axiomatic epistemology-and-axiology systems Σ and Σ+C, which are already published in [Lobovikov, 2020] and, [Lobovikov, 2021], respectively. However, I think that for realizing the goal of the paper, Σ is not quite sufficient, and Σ+C is not optimal. For the optimization, it is worth adding to Σ not only the *modality of consistency* (the first "C", which has been added to Σ in Σ+C), but also the *modality of completeness* (the second "C", which is to be added to Σ+C in Σ+2C submitted below in this article for the first time).

In this paper, the above-mentioned significantly *novel mutation* of the logically formalized *multimodal* axiomatic epistemology system Σ is to be used for logical analyzing a system of philosophical foundations of mathematics which (system) is made up by the following set of statements ST1 – ST8:

ST1: proper mathematical knowledge of ω is *a-priori* one. See, for instance, [Kant, 1994, p. 16, 18].

ST2: truth of ω and provability of ω are logically equivalent in the rationalistic optimism *ideal* created by G. W. Leibniz [1903; 1969; 1981] and D. Hilbert [1990; 1996a –1996c]. Of the rationalistic optimism *ideal* and K. Gödel's philosophy see also [Ершов и Целищев, 2012], [Целищев, 2013], [Zach, 2019]. By the way, here it is relevant to note that there is a nontrivial *formal-axiological* equivalence of "true" and "provable" [Lobovikov, 2009] but the almost unknown "*formal-axiological* equivalence" and the well-known "formal-logical one" are not identical.

ST3: consistency of proper mathematical statement or theory ω and provability of consistency of ω are logically equivalent in Hilbert's *ideal* of self-sufficient (self-dependent) mathematics;

ST4: truth of ω and consistency of ω are logically equivalent (in the ideal).

ST5: truth of ω is logically equivalent to ω (in the ideal).

ST6: consistency of ω is logically equivalent to completeness of ω (in the ideal).

ST7: truth of ω and completeness of ω are logically equivalent (in the ideal).

ST8: completeness of proper mathematical statement or theory ω and provability of completeness of ω in a consistent theory are logically equivalent in the *ideal* of self-sufficient (self-dependent) mathematics;

D. Hilbert was not alone; his rationalistic optimism *ideal* (norm) of mathematical activity was attractive also for A. Tarski and for many other prominent mathematicians. Even being aware of Gödel's theorems of incompleteness, A. Tarski believed and wrote that *it is good (desirable)* for a mathematician to prove that ST2 is true in relation to a concrete mathematical statement or theory ω, if this proving is possible [1948, pp. 185–189]. Also being aware of Gödel's theorems of incompleteness and taking them into an account, V. V. Tselishchev writes (in perfect accordance with Tarski) that proving consistency and completeness is a *norm (duty)* which is *prescribed (obligatory)* for a mathematician, if such proving is possible [Целищев, 2004a; 2004b; 2005]. Here the famous bimodal Kant-principle "obligation (duty) implies possibility" ($Op \supset \Diamond p$) works. As due to the theorems by Gödel, proving completeness of the formal arithmetic system (under the condition of its consistency) is impossible, there is no violation of the *norm* (the relevant *obligation* is abolished by *modus tollens*).

If Hilbert's formalism ideal and program of/for philosophical grounding mathematics was fulfilled (i.e. if the ideal created by him was realized), then the system of mathematical knowledge (as a whole) would be *self-sufficient (self-dependent)* one. Unfortunately, today there is a widespread opinion (a *statistical norm* of thinking and affirming) that Hilbert's ideal and the formalism program targeted at realizing this ideal were totally annihilated by Gödel's theorems of incompleteness. However, the widespread opinion is not able to explain the reason (philosophical foundation) of/for Hilbert's creating the ideal and the formalism program in question. The folks talking of Gödel's termination of Hilbert's formalism program do not recognize a possibility of existence of a *not empty* domain in which Hilbert's ideal and the formalism program targeted at realizing this ideal are perfectly adequate even today (and forever). If so, then significance of Gödel's famous results is reduced to significance of precise limiting the mentioned *not empty* domain, i.e. to significance of establishing quite exact border-lines of/for that domain. The present paper is aimed at recognizing and explicating the strong *reason* of/for Hilbert's creating the formalism program and at giving an exact definition of the realm of the program's soundness missed by the mentioned folks.

By analyzing the above statements ST1 – ST8, it is possible to focus on the set of *qualitatively different* modalities which are indispensable for formulating ST1 – ST8, namely the following: "*knows* that…"; "*a-priori knows* that…"; "*empirically knows* that…"; "*it is true* that…"; "it is *provable* in a consistent theory that…", "it is *consistent* that…", "it is *complete* that…". The first five modalities are taken into an account by Σ while the last two ones are not. Therefore, successfully to cope with realizing the research goal, it is worth making a mutation in Σ+C by adding the novel modality "*Completeness*" to it. In Σ+C, the symbol Cω stands for "it is *consistent* that ω". As now the novel modality "Completeness" is added to Σ+C, let the symbol "Σ+2C" be the name of/for the result of adding the two modalities (Consistency and Completeness) to Σ. Thus, the general idea of this article is introduced in first approximation which is sufficient to begin with. Now let us move to the next paragraph giving a precise definition of the multimodal formal axiomatic system Σ+2C to be used in this paper as an effective means of/for realizing the goal.

2. A New Formal Multimodal Axiomatic Epistemology-and-Axiology Theory Σ+2C

The axiomatic system Σ-2C is a result of developing further the formal axiomatic *epistemology-and-axiology* theory Σ [Lobovikov, 2020] and the formal axiomatic *epistemology-and-axiology* theory Σ+C [Lobovikov, 2021]. The synthesizing compound term "*epistemology-and-axiology*" is odd (unhabitual); moreover, it is a manifest challenge to the faith of positivist-minded philosophers in absolute universality of the "logically unbridgeable gap between statements of being and corresponding ones of value or duty". According to R. Carnap [1931; 1935; 1956; 1967] and his adherents, sentences of natural language of theology, axiology and metaphysics are meaningless; they are to be eliminated from human culture (especially from science) by logical analysis. Today, such an unrealistic view of the extremist-minded positivists is not very popular. In our time, a much more sophisticated view of complicated relationship between logic and metaphysics has been developed. In this relation, see, for instance, the monograph [Целищев, 2021]. The *epistemology-and-axiology* theories Σ, Σ+C, and Σ-2C are representations of *exactly synthesizing* approach to logic and metaphysics: in these theories, multimodal metaphysics and axiology are logically formalized; philosophical ontology and universal epistemology are united with formal axiology. In the given paper, the outcome of such uniting is applied to philosophical grounding mathematics.

To construct a perfectly exact definition of the formal axiomatic theory Σ-2C, it is necessary to begin with manifest giving precise definitions of the notions: "*alphabet* of object-language of Σ-2C"; "*term* of Σ-2C"; "*formula* of Σ-2C"; "*axiom* of Σ-2C". Strict definitions of these notions of Σ-2C *look similar* to the definitions of corresponding notions of Σ and "Σ+C", which are already published (open access) in [Lobovikov, 2020] and [Lobovikov, 2021], respectively. Nevertheless, strictly speaking, in this article, it is quite indispensable to construct precise definitions of "*alphabet* of object-language of Σ-2C", "*term* of Σ-2C", "*formula* of Σ-2C", and "*axiom* of Σ-2C", in spite of the mentioned similarity, as *similarity is not logically equivalent to identity*; the relevant notions of Σ and "Σ+C" are not identical to the corresponding similar notions of Σ-2C. Therefore, let us start precise formulating the definitions quite indispensable for perfect understanding this article in spite of the false impression (illusion) that they are repetitions of the already published statements. Let us begin with precise defining the notion "alphabet of object-language of formal theory Σ-2C".

By definition, the alphabet of object-language of formal theory Σ-2C contains all the signs which belong to the alphabet of object-language of formal theory Σ. But the conversion of this sentence is not true, as, in Σ-2C, some important new symbols are added to the alphabet of Σ and to the alphabet of Σ+C. The outcome of these significant mutations (additions) is the following exact definition of the alphabet of object-language of Σ-2C.

1) The lowercase Latin letters p, q, d (and these letters having lower number indexes) belong to the alphabet of object-language of Σ-2C. Such and only such lowercase Latin letters are named "*dictum variables*". In the alphabet of object-language of Σ-2C, *not all lowercase Latin letters are called dictum variables* because, according to the given definition, those lowercase Latin letters which belong to the set {g, b, e, n, x, y, z, a, s, h, t, f} do not belong to the set of *dictum variables* of object-language of Σ-2C.

2) The lowercase Latin letters a, s, h (and these letters having lower literal indexes: a_t, s_m, h_s) belong to the alphabet of object-language of Σ-2C. Such and only such lowercase Latin letters are named "*dictum constants*".

3) The habitual logic symbols $\neg, \supset, \leftrightarrow, \&, \vee$ named, respectively, "classical negation", "classical (or 'material') implication", "classical equivalence", "classical conjunction", "classical not-excluding disjunction" belong to the alphabet of object-language of Σ-2C.

4) Elements of the set {□, K, A, E, S, T, F, P, D, C, Y, G, W, O, B, U, J}, containing □ and some (but not all) capital Latin letters having no indexes, are elements of the alphabet of object-language of Σ-2C. These elements of the alphabet are named "modality symbols" in Σ-2C.

5) The lowercase Latin letters *x*, *y*, *z* (and also these letters having lower number indexes) belong to the alphabet of object-language of Σ-2C. Such and only such letters are named "*axiological variables*" in Σ-2C.

6) The lowercase Latin letters "g" and "b" named "*axiological constants*" also are elements of the alphabet of object-language of Σ-2C.

7) The capital Latin letters having number indexes – E^1, C^1, K^1, K^2, E^2, C^2, C_j^n, B_i^n, D_m^n, A_k^n, … are elements of the alphabet of object-language of Σ-2C. Such capital Latin letters are named "*axiological-value-functional symbols*". Here the upper number index *n* informs that the indexed axiological-value-functional symbol is *n*-placed one. The axiological-value-functional symbols may possess no lower number index. But, if value-functional symbols possess lower number indexes, then, if these indexes are different, then the indexed functional symbols are different ones.

8) The signs "(" and ")" named "round brackets" are elements of the alphabet of object-language of Σ-2C as well. These auxiliary signs are utilized in the present article as usually in symbolic logic, namely, as pure technical symbols.

9) The signs "[" and "]" ("square brackets") are elements of the alphabet of object-language of Σ-2C also. However, it is worth emphasizing here that in contrast to the "round brackets", in Σ-2C, the "square brackets" are used not as the habitual pure technical symbols, but as *ontologically meaningful* signs. Such nonstandard using the "square brackets" is psychologically unexpected (unhabitual) one. In relation to natural language psychology, square brackets and round ones seem identical as very often in natural language they are used as synonyms. But in the object language of Σ-2C, the two kinds of brackets possess *significantly different* meanings (play substantially different roles): usage of round brackets is purely technical (auxiliary) one, while square-bracketing possesses an *ontological* meaning. The *ontological* meaning of square-bracketing is defined below in that part of the present paper which is devoted to *semantics* of object-language of Σ-2C. Nevertheless, even at the level of syntaxis of the artificial object language of Σ-2C, square brackets *play a substantial role* in the precise definition of the concept "formula of Σ-2C". (This definition is to be given below in this section of the article.) Moreover, square-bracketing *plays a substantial role* in the precise formulations of some axiom-schemes of Σ-2C" (which formulation are to be given below also in this section of the article).

10) An unhabitual artificial symbol "=+=" named "*formal-axiological equivalence*" is an element of the alphabet of object-language of Σ-2C. The odd symbol "=+=" *plays a substantial role* in the precise definition of the concept "formula of Σ-2C" and also in the precise formulations of some axiom-schemes of Σ-2C.

11) A sign is an element of the alphabet of object-language of Σ-2C, if and only if the sign belongs to this alphabet due to the above-formulated items 1) – 10) of the given definition.

Any finite chain (queue) of symbols is named "an *expression* of the object-language of Σ-2C", then and only then, when that chain contains such and only such signs which are elements of the alphabet of object-language of Σ-2C.

A precise definition of the concept "*term* of Σ-2C" is the following.

1) the above-mentioned *axiological variables* (see the definition of alphabet of Σ-2C) are terms of Σ-2C.

2) the above-mentioned *axiological constants* (see the definition of alphabet of Σ-2C) are terms of Σ-2C.

3) If Φ_k^n is an *n-placed axiological-value-functional symbol* (see the definition of alphabet of Σ-2C), and $t_i, \dots t_n$ are *terms* of Σ-2C, then $\Phi_k^n t_i, \dots t_n$ is a term of Σ-2C. (It is worth noting here that signs $t_i, \dots t_n$ belong to the meta-language; because they denote *any* terms of Σ-2C; the analogous note is worth making with respect to the sign Φ_k^n belonging to the meta-language as well.)

4) An expression of the object-language of Σ-2C is a term of Σ-2C, then and only then, when it is so due to the above-formulated items 1) – 3) of the given definition.

Thus, the *syntaxis* aspect of the abstract notion "*term* of Σ-2C" is quite fixed. Now we are to move to constructing exact definition of the *syntaxis* aspect of the abstract notion "*formula* of Σ-2C". To perform this move, let us accept the convention that in the given article, lowercase Greek letters α, β, and ω (belonging to meta-language) denote *any* formulae of Σ-2C. Keeping this convention in mind, it is possible to give the following precise definition of the notion "formula of Σ-2C".

1) All the lowercase Latin letters which are above-called "*dictum variables*" and all the lowercase Latin letters which are above-called "*dictum constants*" belong to the set of formulae of Σ-2C.

2) When α and β are formulae of Σ-2C, then all the expressions of the object-language of Σ-2C, which (expressions) have forms $\neg\alpha$, $(\alpha \leftrightarrow \beta)$, $(\alpha \supset \beta)$, $(\alpha \vee \beta)$, $(\alpha \,\&\, \beta)$, belong to the set of formulae of Σ-2C as well.

3) When t_i and t_k are terms of Σ-2C, then $(t_i =+= t_k)$ is a formula of Σ-2C.

4) When t_i is a term of Σ-2C, then $[t_i]$ is a formula of Σ-2C.

5) When α is a formula of Σ-2C, and the symbol Ψ (belonging to the meta-language) denotes any modality symbol from the set of {□, K, A, E, S, T, F, P, D, C, Y, G, W, O, B, U, J}, then any expression of object-language of Σ-2C having the form Ψα, is a formula of Σ-2C also. It is worth noting here, that, strictly speaking, the expression Ψα (belonging to the meta-language) is not a formula of Σ-2C, but a scheme of formulae of Σ-2C.

6) Chains of symbols from the alphabet of object-language of Σ-2C are formulae of Σ-2C, if and only if it is so due to the items 1) – 5) of the given definition.

In this part of the article which (part) is reduced intentionally to *syntaxis* of object-language of *multimodal* formal theory Σ-2C, the set of modality symbols {□, K, E, A, S, T, F, P, D, C, Y, G, W, O, B, U, J} is nothing but a set of very short *names*. The symbol □ is a name for the alethic modality "it is *necessary* that …". The symbols K, E, A, S, T, F, P, D, C, respectively, are names of/for the modal expressions "agent *Knows* that…", "agent *Empirically* (*a-posteriori) knows* that…", "agent *A-priori knows* that…", "under some concrete conditions in some definite time-and-space, an agent has a *Sensation, i.e. verification by feeling* (either immediately or by means of mediating tools), that…", "it is *True* that…", "agent has *Faith* that… (or agent believes that…)", "it is *Provable* in a consistent theory that…", "there is *an algorithm for Deciding* that… (hence, a machine could be constructed for such Deciding)", "it is *Consistent* that…", "it is *Complete* that …".

The symbols G, W, O, B, U, J, respectively, are names of/for the modal expressions "it is *Good* (morally perfect) that…", "it is *Wicked* (morally bad, imperfect) that…", "it is *Obligatory* (mandatory, compulsory) that …", "it is *Beautiful* (aesthetically perfect) that …", "it is *Useful* (helpful, valuable, gainful, rewarding) that …", "it is a *Joy* (happiness, pleasure, delight) that …". In the present section of the article, pure syntaxis meanings of the modal symbols are defined quite precisely (although not manifestly but indirectly) by the below-given schemes of own (proper) axioms of multimodal formal philosophy (epistemology-and-axiology) system Σ-2C.

The proper (own) axioms of multimodal formal philosophy are added to proper (pure) logic axioms which are essentially *similar* to the ones of classical logic of propositions. Thus, proper formal logic axioms and formal logic inference rules of Σ, Σ+C, and Σ-2C are *analogous* to the ones of classical sentential logic calculus. As the reference to the *similarity (analogy)* is not a perfect definition, below I am to define the set of proper formal logic axioms of Σ-2C rigorously in the following way. If α, β, and ω are formulae of Σ-2C, then the below-located schemes of formulae of Σ-2C are schemes of proper (pure) logic axioms of Σ-2C.

PLA-1: $\alpha \supset (\beta \supset \alpha)$.

PLA-2: $(\alpha \supset (\beta \supset \omega)) \supset ((\alpha \supset \beta) \supset (\alpha \supset \omega))$.

PLA-3: $(\neg\alpha \supset \beta) \supset ((\neg\alpha \supset \neg\beta) \supset \alpha)$.

By definition, in Σ-2C, there is a formal logic derivation rule called "MP (*modus ponens*)": If α and β are formulae of Σ-2C, then $\alpha, (\alpha \supset \beta) \mid\!- \beta$. (Here the symbol "… |– …" stands for "in Σ-2C, from … it is formally logically derivable that…".) Although in the definition of Σ-2C, only one formal logic derivation rule is mentioned manifestly, it is possible to infer deductively and use systematically also any *derivative* inference-rules justifiable in classical propositional logic.

It is worth highlighting here that, strictly speaking, PLA-1, PLA-2, PLA-3, are results of a substantial *mutation* (heuristically useful one) in the well-known classical propositional logic axioms; the above-introduced *dictum variables* have replaced the corresponding propositional ones. Concerning the definitions of additional logic connectives, and the logic-inference-rules (*modus ponens* and all derivative rules), the relevant mutation (generalization) is to be taken into an account as well.

The schemes of pure logic axioms PLA-1, PLA-2, PLA-3, the relevant definitions of additional logic connectives, and the logic-inference-rules (*modus ponens* and all *derivative* rules of logic inference) are applicable to all formulae of the multimodal theories Σ+C, and Σ-2C. Hence, the proper logic foundations of Σ+C, and Σ-2C are identical but the mentioned logically formalized axiomatic systems based on these identical logic foundations are different. It seems that, corresponding definitions of Σ, Σ+C, and Σ-2C are identical, but strictly speaking, it only seems so. The formal theories Σ, Σ+C, and Σ-2C have different alphabets of their object-languages, different sets of expressions, different sets of terms, different sets of formulae, different sets of definitions, different sets of axioms, and, finally, different sets of theorems.

In the given section of the article, exactly *syntax* meanings of all the modality symbols and of all the other special signs included into the alphabet of object language of Σ-2C are defined precisely by the following list of schemes of proper philosophical (epistemological and axiological) axioms of Σ-2C. (Certainly, such *axiomatic* definition of proper epistemology-and-axiology notions is *not manifest* one, but, nevertheless, it is *quite precise* one.) If α, β, ω are any formulae of Σ-2C, then any such and only such expressions of the object language of Σ-2C, which have the following forms, are *proper axioms* of Σ-2C.

Axiom scheme AX-1: $A\alpha \supset (\Box\beta \supset \beta)$.

Axiom scheme AX-2: $A\alpha \supset (\Box(\omega \supset \beta) \supset (\Box\omega \supset \Box\beta))$.

Axiom scheme AX-3: $A\alpha \leftrightarrow (K\alpha \,\&\, (\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta)))$.

Axiom scheme AX-4: $E\alpha \leftrightarrow (K\alpha \,\&\, (\Diamond\neg\alpha \vee \Diamond S\alpha \vee \neg\Box(\beta \leftrightarrow \Omega\beta)))$.

Axiom scheme AX-5: $\Omega\alpha \supset \Diamond\alpha$. (This is a substantial *multimodal generalization* of "Kant principle" combining the deontic and the alethic modalities: $O\alpha \supset \Diamond\alpha$.)

Axiom scheme AX-6: $(\Box\beta \,\&\, \Box\Omega\beta) \supset \beta$. (This is a substantial *multimodal generalization* of the selebrated formula $(\Box\beta \supset \beta)$ underivable in Σ-2C. Concerning the underivability of $(\Box\beta \supset \beta)$, see [Lobovikov, 2018].

Axiom scheme AX-7: $(t_i =+= t_k) \leftrightarrow (G[t_i] \leftrightarrow G[t_k])$.

Axiom scheme AX-8: $(t_i =+= g) \supset \Box G[t_i]$.

Axiom scheme AX-9: $(t_i =+= b) \supset \Box W[t_i]$.

Axiom scheme AX-10: $(G\alpha \supset \neg W\alpha)$. See the monograph by A.A. Ivin [1970].

Axiom scheme AX-11: $(W\alpha \supset \neg G\alpha)$. See A.A. Ivin's book [1970].

Definition scheme DF-1: when ω is a formula of Σ-2C, then $\Diamond\omega$ is a *name* of/for $\neg\Box\neg\omega$.

In AX-3, AX-4, AX-5, and AX-6, the symbol Ω (belonging to the meta-language) stands only for a (any) "*perfection* modality". Not all the above-mentioned modalities are called "*perfection* ones". The set Δ of signs denoting *perfection*-modalities (or simply, "perfections") is the following {K, D, F, C, Y, P, J, T, B, G, U, O, □}. Obviously, Δ is only a subset of the set of all signs denoting modalities taken into an account in this article. For instance, W and ◊ are names of/for modalities which are not perfections. Including C and Y into the set Δ of *perfection*-modalities is quite natural as "consistency" and "completeness" are important *perfections* of a theoretical system [Тарский, 1948, pp. 185, 186]. As a rule, *de-dicto*-modalities are attached to a *dictum*. Usually, the word "*dictum*" is translated (interpreted) from the Latin language as a "*proposition* (or sentence)", but, in principle, it is possible to *generalize* the habitual meaning of the word "*dictum*" in such a way that a theoretical (deductive) system would be a *dictum* as well. Dictum is "what is affirmed (stated)" but a theory also can be"what is affirmed (stated)".

A justification of AX-10 and AX-11 can be found in the monograph [Ивин, 1970] devoted to formal logic of evaluations. But the almost unknown (aunhabitual) axiom-schemes AX-7, AX-8, and AX-9 represent not the formal logic but a formal axiology (universal theory of abstract value forms). The notion "formal logic" is not logically equivalent to the notion "formal axiology", consequently, "formal-logic inconsistency" and "formal-axiological one" are not synonyms. The significant logic-difference between notions "*formal-axiological* contradiction" and "formal-logic one" explains a psychologically unexpected possibility of deductive proof of the *formal-axiological inconsistency* of the formal arithmetic theory [Lobovikov, 2011; Лобовиков, 2011].

Certainly, the above-given exact *syntactic* definitions are *semantically meaningless*; this is not a contingent omission by negligence but such a deliberately accepted scientific abstraction which is quite resonable within an adequately defined domain. Threfore, now, to make the article perfectly meaningful one, it is indispensable to move directly to *semantics* of the language of Σ-2C.

3. Defining semantics of/for the multimodal formal axiomatic theory Σ +2C

In the above section 2 of the article, the purely syntactic definition of Σ+2C has been presented; it has been deliberately deprived of its proper philosophical contents (owing to the relevant abstraction). The formal axiomatic theory Σ+2C is a *multimodal* one, but up to the present moment concrete contents of the modalities have been exposed not sufficiently; the theory Σ+2C has been considered as actually formal one. Below in this section of the article, we are to relax the formality of Σ+2C substantially by moving directly to concrete philosophical contents of the modalities under consideration in Σ+2C. It is presumed in the present article that the semantic meanings of the proper logic symbols of the artificial language of classical symbolic logic are already well-defined by relevant handbooks. As the semantic meanings of the proper logic symbols are well-known, there is no need to define them here. On the contrary, the extraordinary (very unusual) signs of the artificial language of Σ+2C require a systematical specification of their semantic meanings.

Meanings of the lowercase Latin letters p, q, d (and of these letters having lower number indexes) above-named "*dictum variables*" are analogous to the meanings of the well-known "*propositional variables*". However, there is a significant difference: in Σ+2C, "*dictum variables*" take their values from the set of dictums to which set not only all true or false propositions but also all true or false theories belong. Thus, the dictum variables range over the set of dictums which are either true or false. If an interpretation of Σ+2C is given, then a *dictum constant* means (in that interpretation) quite a definite (fixed) element of the set of dictums, i.e. either a concrete true or false proposition or a concrete true or false theory.

As a habit (custom) rule (statistical norm), the word "*dictum*" is translated (interpreted) from the Latin language as "affirmation of (a sentence…)" or "expression of (a thought …) in words". However, there is a possibility to move from "affirmation of (a sentence…)" to "affirmation of (a sentence …, or a theory …)" as a theory is also something what can be affirmed. In the given article, it is presumed that a theory is a dictum as well. Thus, attaching *de-dicto*-modalities C (consistency) and Y (completeness) to theories is vindicated in Σ+2C. Concerning general philosophical investigations of modalities *de-dicto* and *de-re*, see, for instance, [Prior, 1952], [Kneale, 1962], [Sosa, 1970], [Chisholm, 1976], [Целищев, 1978], [Salmon, 1997].

Defining semantic meanings is defining an *interpretation-function*. To define the interpretation-function one has to define (1) a set which plays the role of "domain (or field) of interpretation" (let the interpretation-domain be denoted by the letter M) and (2) a "valuator (evaluator)" *V*. By definition, in a standard interpretation of Σ+2C, M is such a set, every element of which has: (1) one and only one *axiological value* from the set {good, bad}; (2) one and only one *ontological value* from the set {exists, not-exists}.

The *axiological variables* (z, x, y, z_i, x_k, y_m) take their values from the set M.

The *axiological constants* "b" and "g" mean "bad" and "good", respectively.

Valuating an element from M by a concrete (fixed) interpreter V is ascribing an *axiological value* (either good or bad) to that element. The interpreter V may be either collective or individual one. Certainly, a change of V can change some relative evaluations, but cannot change the set of laws of two-valued algebra of formal axiology which are not relative but absolute evaluations, namely, such and only such *constant valuation-functions* which have the value g (good) under any possible combination of axiological values of their axiological variables. Although V is a variable taking its values from the set of all possible interpreters, a perfectly defined interpretation of Σ+2C necessarily implies that the value of V is fixed. A change of V necessarily implies a change of interpretation.

In the present article, "e" and "n" stand for "… exists" and "… not-exists", respectively. The signs "e" and "n" are named "*ontological constants*". By definition, in a standard interpretation of Σ+2C, one and only one element of the set {{g, e}, {g, n}, {b, e}, {b, n}} corresponds to every element of M. The signs "e" and "n" belong to the meta-language. By definition of the alphabet of object-language of Σ+2C, "e" and "n" do not belong to the object-language. Nevertheless, "e" and "n" are *indirectly* represented at the level of object-language of Σ+2C by means of *square-bracketing*: "t_i exists" is represented by [t_i]; "t_i does not exist" is represented by ¬[t_i]. This means that square-bracketing is a significant part of exact defining formal-axiological-and-ontological semantics of Σ+2C.

N-placed terms of Σ+2C are interpreted as *n*-placed evaluation-functions defined on the set M. The notion "One-placed evaluation-function" is exemplified below by the **Table 1**. (It is relevant to recall here that the upper index 1 standing immediately after a capital letter means that this letter stands for a one-placed evaluation-function.)

Table 1. Definition of the evaluation-functions determined by one evaluation-argument

x	$B_1{}^1x$	$N_1{}^1x$	$C_1{}^1x$	$I_1{}^1x$	$Z_1{}^1x$	$S_1{}^1x$	$U_1{}^1x$	$A_1{}^1x$	$G_1{}^1x$	$P_1{}^1x$	$H_1{}^1x$	$R_1{}^1x$	$C_2{}^1x$	$I_2{}^1x$
g	g	b	g	b	b	b	b	g	g	g	b	b	g	b
b	b	g	b	g	b	b	b	g	g	b	g	g	b	g

In the **Table 1**, the one-placed term $B_1{}^1x$ is interpreted as *one-placed evaluation-function* "*being (existence) of* (what, whom) *x*"; the term $N_1{}^1x$ is interpreted as *evaluation-function* "*non-being (nonexistence) of* (what, whom) *x*". $C_1{}^1x$ – "*consistency of* (what, whom) *x*". $I_1{}^1x$ – "*inconsistency of* (what, whom) *x*". $Z_1{}^1x$ – "*formal-axiological inconsistency (or absolute inconsistency) of* (what, whom) *x*". $S_1{}^1x$ – "*x*'s *formal-axiological self-contradiction*". $U_1{}^1x$ – "*absolute non-being of* (what, whom) *x*". $A_1{}^1x$ – "*absolute being of* (what, whom) *x*". $G_1{}^1x$ – "*absolute goodness of* (what, whom) *x*", or "*absolute good* (what, who) *x*". $P_1{}^1x$ – "*positive evaluation of* (what, whom) *x*". $H_1{}^1x$ – "*negative evaluation of* (what, whom) *x*". $R_1{}^1x$ – "*resistance to* (what, whom) *x*". $C_2{}^1x$ – "*completeness of* (what, whom) *x*". $I_2{}^1x$ – "*incompleteness of* (what, whom) *x*".

The notion "*two-placed evaluation-function*" is instantiated by the below **Table 2**. (In this paper, the upper index 2 standing immediately after a capital letter means that this letter stands for *a two-placed function*; difference of lower number-indexes means difference of the relevant symbols, for example, in the above-presented **Table 1**, $C_1{}^1x$ and $C_2{}^1x$ are different symbols.)

Table 2. Definition of the evaluation-functions determined by two arguments

x	y	K^2xy	S^2xy	X^2xy	T^2xy	Z^2xy	P^2xy	C^2xy	E^2xy	V^2xy	N^2xy	Y^2xy
g	g	g	b	b	b	b	g	g	g	b	b	g
g	b	b	g	b	b	b	g	b	b	g	b	g
b	g	b	g	g	g	g	b	g	b	g	b	g
b	b	b	g	b	b	b	g	g	g	b	g	b

In the **Table 2**, the two-placed term K^2xy is interpreted as evaluation-function "*being of both x* and *y* together", or "*joint being* of *x* with *y*". S^2xy is interpreted as "*separation, divorcement* between *x* and *y*. The term X^2xy – evaluation-function "*y's being without x*", or "joint being of *y* with nonbeing of *x*". T^2xy – "*termination* of *x* by *y*". Z^2xy – "*y*'s *contradiction* to (with) *x*". P^2xy – "*preservation, conservation, protection* of *x* by *y*". C^2xy is interpreted as evaluation-function "*y's existence, presence in x*". E^2xy – "*equivalence, identity (of values)* of *x* and *y*". V^2xy – "*choosing and realizing* such and *only such* an element of the set {*x, y*}, which is: 1) *the best* one, if both *x* and *y* are good; 2) the least bad one, if both *x* and *y* are bad; 3) the good one, if *x* and y have opposite values. (Thus, V^2xy means an *excluding choice and realization of only the optimal* between *x* and *y*.) The term N^2xy is interpreted as evaluation-function "*realizing neither x nor y*". Y^2xy is interpreted as evaluation-function "realizing a *not-excluding-choice* result, i.e. 1) *realizing* K^2xy *if both x and y are good*, and 2) *realizing* V^2xy *otherwise*". Additional exemplifications of "*two-placed evaluation-function*" may be found in [Лобовиков, 2018; Lobovikov, 2020; 2021].

To exclude possibilities of misunderstanding the present article, here it is quite relevant to highlight that in a standard interpretation of Σ+2C, the signs B_1^1x, N_1^1x, C_1^1x, K^2xy, C^2xy, E^2xy, V^2xy stand not for predicates but for *n*-placed *evaluation-functions*. Being given an interpretation of Σ+2C, such expressions of the object-language of Σ+2C, which have forms (t_i=+=t_k), (t_i=+=g), (t_i=+=b), are representations of *predicates* in Σ+2C.

By definition of semantics of Σ+2C, if t_i is a term of Σ+2C, then, being interpreted, such a formula of Σ+2C, which has the form [t_i], is an *either true or false proposition* "t_i exists". Thus, by definition, in a standard interpretation, formula [t_i] is true if and only if t_i has the *ontological value* "e (exists)" in that interpretation. Also, by definition, the formula [t_i] is false in a standard interpretation of Σ+2C, if and only if t_i has the ontological value "n (not-exists)" in that interpretation.

By definition of semantics of Σ+2C, in a standard interpretation of Σ+2C, the formula scheme (t_i =+= t_k) is a proposition possessing the form "t_i is *formally-axiologically equivalent* to t_k"; this proposition is true if and only if (in that interpretation) the terms t_i and t_k obtain identical *axiological values (*from the set {good, bad}) under any possible combination of *axiological values* of their *axiological variables*.

By definition of semantics of Σ+2C, in a standard interpretation of Σ+2C, the formula scheme (t_i =+= b) is a proposition having the form "t_i is a *formal-axiological contradiction*" (or "t_i is *formally-axiologically, or invariantly, or absolutely bad*"); this proposition is true if and only if (in that interpretation) the term t_i acquires axiological value "bad" under any possible combination of axiological values of the axiological variables.

By definition of semantics of Σ+2C, in a standard interpretation of Σ+2C, the formula scheme (t_i =+= g) is a proposition having the form "t_i is a *formal-axiological law*" (or "t_i is *formally-axiologically, or invariantly, or absolutely good*"); this proposition is true if and only if (in the interpretation) the term t_i acquires *axiological value* "good" under any possible combination of axiological values of the axiological variables.

In respect to the above-given definition of sematic meaning of (t_i =+= t_k) in Σ+2C, it is indispensable to highlight the important linguistic fact of homonymy of the words "is", "means", "implies", "entails", "equivalence" in natural language. On the one hand, in natural language, these words may have the well-known formal logic meanings. On the other hand, in natural language, the same words may stand for the above-defined *formal-axiological-equivalence* relation "=+=". This ambiguity of natural lenguage is to be taken into an account; the different meanings of the homonyms are to be separated systematically; otherwise the homonymy can head to logic-linguistic illusions of paradoxes.

Owing to the above-presented definition of formal-axiological-and-ontological semantics of Σ+2C, it is easy to recognize that the two-valued algebraic system of formal axiology is nothing but abstract *theory-of-relativity* of evaluations; in this theory-of-relativity, the *formal-axiological laws* (constantly good evaluation-functions) of that algebraic system are *invariants* in relation to all possible transformations of interpreter V.

Thus, although it is an indisputable (perfectly evident) fact that relativity (and mutability) of empirical valuations does exist, the valuation-invariants (immutable universal laws of valuation-relativity) do exist as well [Lobovikov, 2020].

4. *Some curious properties of the logically formalized axiomatic epistemology-and-axiology system* Σ*+2C*

Certainly, Σ+2C is *not a normal* modal logic system in that special meaning of the term "*normal* modal logic" which is precisely defined in [Kripke, 1963; 1965] and [Bull and Segerberg, 1984]. This is so because the formula-schemes ($\Box\beta \supset \beta$) and ($\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$) are not provable in Σ+2C, and Gödel's necessitation rule is not provable in Σ+2C as well. However, it is possible easily to obtain a *normal* modal logic system by *modus ponens* in result of adding the assumption Aα to Σ+2C. Nevertheless, according to Σ+2C, a discourse of *empirical* knowledge and of *knowledge in general* is beyond the domain of relevant applicability of the *normal* modal logic.

Generally speaking, the wonderful formula-scheme ($\alpha \leftrightarrow T\alpha$) is not provable in Σ+2C, However, adding the assumption Aα to Σ+2C makes a system (let us call it "Σ+2C +Aα"), in which ($\alpha \leftrightarrow T\alpha$) is a provable formula-scheme.

Concerning *epistemic* modal logic [Hintikka, 1962; 1974], [Hintikka and Hintikka, 1989], it is worth mentioning here that the well-known formula ($Kq \supset q$) is not provable in Σ+2C. Being accepted as a *strictly universal* principle, ($Kq \supset q$) contradicts to the *evolutionary* epistemology, in which belief revision and *knowledge revision* are taken seriously [Bradie and Harms, 2020], [Лобовиков, 2018]. Thus, Σ+2C is a more realistic model of reasoning of *empirical* knowledge and of *knowledge in general* (in comparison with the epistemologies accepting ($Kq \supset q$) as a *necessarily universal* principle). Relativity, flexibility, and flow of *empirical* knowledge do exist. This is a fact of history of cognition. Consequently, if accepting the *evolutionary* epistemology is rational then habitual accepting the formula ($Kq \supset q$) as a theorem of epistemic logic is not rational. Taking the *evolutionary* epistemology seriously necessitates that, *in general, modal logic of knowledge is not-normal one* [Лобовиков, 2017; 2018]. According to Σ+2C, the *normal* modal logic of knowledge is quite rational only in such a very rare (extraordinary) *particular* case when the assumption Aα is true. Although ($Kq \supset q$) is not a theorem in Σ+2C, a less strong formula ($Kq \supset \Diamond q$) is derivable inΣ+2C. It is a significant difference between Σ and Σ+2C, that formulae ($Oq \supset \Diamond q$), ($Gq \supset \Diamond q$), ($Tq \supset \Diamond q$), ($Pq \supset \Diamond q$), which are important for philosophy, are formally derivable in Σ+2C (from the axiom-scheme AX-5 by relevant substitutions) but not derivable in Σ. One of the benefits and the novelties of the particular system Σ+2C originally constructed in the present article is justification of the nontrivial bimodal philosopphical principle called "Kant principle", which is modeled by the theorem ($Op \supset \Diamond p$). (Here I shall abstain from discussing the problem of I. Kant's authorship of this bimodal principle of ethics and jurisprudence as it would be a deviation from the main theme and goal of the article.) Another important benefit (and significant novelty) of constructing the original system Σ+2C is represented in the following section of the paper. Exactly this concrete benefit (and novelty) is my main concern in the given paper.

5. *Some philosophically nontrivial results of applying Σ+2C to uniting K. Gödel's incompleteness theorems with the doctrines by I. Kant and D. Hilbert concerning proper mathematical knowledge system*

Now let us apply the hitherto never considered axiomatic system Σ+2C to the set of statements ST1 – ST8 formulated in the introduction, which statements model Kant's and Hilbert's philosophies of mathematics. Within Σ+2C, by means of its artificial language, the statements ST1 – ST8 (formulated in the introduction) are represented by the following formulae ST1* – ST8*, respectively. The symbol ω in these formulae is interpreted as either a *proper mathematical statement* or a *proper mathematical theory*; the symbols A, T, P, C, Y, respectively, stand for the modalities "it is *A-priori known* that …", "it is *True* that…", "it is *Provable* in the consistent theory that…", "it is *Consistent* that…", "it is *Complete* that…".

ST1*: $A\omega$.

ST2*: $(T\omega \leftrightarrow P\omega)$.

ST3*: $(C\omega \leftrightarrow PC\omega)$.

ST4*: $(T\omega \leftrightarrow C\omega)$.

ST5*: $(T\omega \leftrightarrow \omega)$.

ST6*: $(C\omega \leftrightarrow Y\omega)$.

ST7*: $(T\omega \leftrightarrow Y\omega)$.

ST8*: $(Y\omega \leftrightarrow PY\omega)$.

Now let us prove the following theorem schemes of Σ+2C.

Statement-I: $(A\omega \supset (T\omega \leftrightarrow \omega))$; $(A\omega \supset (T\omega \leftrightarrow P\omega))$; $(A\omega \supset (T\omega \leftrightarrow C\omega))$; $\big(A\omega \supset (T\omega \leftrightarrow D\omega)\big)$; $\big(A\omega \supset (P\omega \leftrightarrow D\omega)\big)$; $\big(A\omega \supset (\omega \leftrightarrow P\omega)\big)$; $\big(A\omega \supset (\omega \leftrightarrow D\omega)\big)$; $\big(A\omega \supset (C\omega \leftrightarrow Y\omega)\big)$; $(A\omega \supset (T\omega \leftrightarrow Y\omega))$. The below chain of formula-schemes is a formal proof of the **Statement-I**.

1. $A\alpha \leftrightarrow (K\alpha \ \&\ (\neg\Diamond\neg\alpha \ \&\ \neg\Diamond S\alpha \ \&\ \Box(\beta \leftrightarrow \Omega\beta))$: axiom-scheme AX-3.
2. $A\alpha \supset (K\alpha \ \&\ (\neg\Diamond\neg\alpha \ \&\ \neg\Diamond S\alpha \ \&\ \Box(\beta \leftrightarrow \Omega\beta))$: from 1 by elimination of $\leftrightarrow$.
3. $A\alpha$: assumption.
4. $(K\alpha \ \&\ (\neg\Diamond\neg\alpha \ \&\ \neg\Diamond S\alpha \ \&\ \Box(\beta \leftrightarrow \Omega\beta)$: from 2 and 3 by *modus ponens*.
5. $\Box(\beta \leftrightarrow \Omega\beta)$: from 4 by elimination of &.
6. $(\beta \leftrightarrow \Omega\beta)$: from 3 and 5 by the rule[1] of elimination of $\Box$.
7. $A\alpha \mid\!- (\beta \leftrightarrow \Omega\beta)$: by 1—6.
8. $A\alpha \mid\!- (\beta \leftrightarrow \Xi\beta)$: from 7 by substituting Ξ for Ω.
9. $A\alpha \mid\!- (\Xi\beta \leftrightarrow \beta)$: from 8 by commutativity of $\leftrightarrow$.
10. $A\alpha \mid\!- (\Xi\beta \leftrightarrow \Omega\beta)$: from 9 and 7 by transitivity of $\leftrightarrow$.
11. $A\alpha \mid\!- (\Omega\beta \leftrightarrow \beta)$: from 7 by commutativity of $\leftrightarrow$.
12. $\mid\!- (A\alpha \supset (\Omega\beta \leftrightarrow \beta))$: from 11 by introduction of $\supset$.
13. $\mid\!- (A\omega \supset (T\omega \leftrightarrow \omega))$: from 12 by substituting (T for Ω) and (ω for α and β).
14. $\mid\!- (A\alpha \supset (\Xi\beta \leftrightarrow \Omega\beta))$: from 10 by introduction of $\supset$.

[1] It is formulated as follows: $A\alpha, \Box\beta \mid\!— \beta$.

15. $|\!-(A\omega \supset (T\omega \leftrightarrow P\omega))$: from 14 by substituting: (T for Ξ); (P for Ω); (ω for α and β).
16. $|\!-(A\omega \supset (T\omega \leftrightarrow C\omega))$: from 14 by substituting: (T for Ξ); (C for Ω); (ω for α and β).
17. $|\!-(A\omega \supset (T\omega \leftrightarrow D\omega))$: from 14 by substituting: (T for Ξ); (D for Ω); (ω for α and β).
18. $|\!-(A\omega \supset (P\omega \leftrightarrow D\omega))$: from 14 by substituting: (P for Ξ); (D for Ω); (ω for α and β).
19. $|\!-(A\alpha \supset (\beta \leftrightarrow \Omega\beta))$: from 7 by introduction of $\supset$.
20. $|\!-(A\omega \supset (\omega \leftrightarrow P\omega))$: from 19 by by substituting: (ω for α and β); (P for Ω).
21. $|\!-(A\omega \supset (\omega \leftrightarrow D\omega))$: from 19 by by substituting: (ω for α and β); (D for Ω).
22. $|\!-\big(A\omega \supset (C\omega \leftrightarrow Y\omega)\big)$: from 14 by substituting: (C for Ξ); (Y for Ω); (ω for α and β).
23. $|\!-\big(A\omega \supset (T\omega \leftrightarrow Y\omega)\big)$: from 14 by substituting: (T for Ξ); (Y for Ω); (ω for α and β).

Thus, the proof is finished.

The succession 1—6 is a formal derivation of $(\beta \leftrightarrow \Omega\beta)$ from the assumption $A\alpha$. The succession 1—10 is a formal derivation of $(\Xi\beta \leftrightarrow \Omega\beta)$ from the assumption $A\alpha$. The chain 1—13 is a formal proof of $(A\omega \supset (T\omega \leftrightarrow \omega))$. The chain 1—15 is a formal proof of $(A\omega \supset (T\omega \leftrightarrow P\omega))$. The queue 1—16 is a formal derivation of $(A\omega \supset (T\omega \leftrightarrow C\omega))$. The succession 1—17 is a formal inference of $\big(A\omega \supset (T\omega \leftrightarrow D\omega)\big)$. The chain 1—18 is a formal proof of $\big(A\omega \supset (P\omega \leftrightarrow D\omega)\big)$. The queue 1—22 is a formal proof of $\big(A\omega \supset (C\omega \leftrightarrow Y\omega)\big)$.

Corollary: from conjunction of the **Statement-I** and the *first* theorem of incompleteness by Gödel, it follows logically that $\neg A\omega$.

Statement II: $(A\omega \supset (C\omega \leftrightarrow PC\omega))$; $(A\omega \supset (Y\omega \leftrightarrow PY\omega))$. The proof of this statement is the following chain of schemes of formulae.

1) $A\alpha \,|\!-(\beta \leftrightarrow \Omega\beta)$: by the chain 1—6 in the above-submitted proof of the **Statement -I**.
2) $A\alpha \,|\!-(\Xi\beta \leftrightarrow \Omega\Xi\beta)$: from 1) by substituting $\Xi\beta$ for β.
3) $|\!-(A\alpha \supset (\Xi\beta \leftrightarrow \Omega\Xi\beta)$: from 2) by the rule of introduction of $\supset$.
4) $|\!-(A\omega \supset (C\omega \leftrightarrow PC\omega)$: from 3) by substituting: (C for Ξ); (P for Ω); (ω for α and β).
5) $|\!-(A\omega \supset (Y\omega \leftrightarrow PY\omega)$: from 3) by substituting: (Y for Ξ); (P for Ω); (ω for α and β).

Corollary: from conjunction of the **Statement-II** and the *second* theorem of incompleteness by Gödel, it follows logically that $\neg A\omega$.

The proof is finished.

It is quite natural to encounter a professional mathematician or logician who has a skeptical view of Leibniz's belief (dream) that developing a modal *propositional* logic can help in discussing, precise formulating, and effective solving if not many then at least some of proper philosophical questions. Why exactly this concrete logic, namely, the *propositional one*? Why, for instance, not the *first-order predicate* logic (or some more general, or rich, or mighty logic system)? The questions are quite natural and nontrivial ones. My answer to them is the following. According to the above-proved **Statement I**, within the formal axiomatic epistemology system Sigma+2C, it is formally provable that $(A\omega \supset (T\omega \leftrightarrow D\omega))$, $(A\omega \supset (P\omega \leftrightarrow D\omega))$. In the standard epistemological interpretation, the theorem-schemes $(A\omega \supset (T\omega \leftrightarrow D\omega))$, $(A\omega \supset (P\omega \leftrightarrow D\omega))$ mean that *a-priori* knowledge system is *decidable*. As it is well-known that the *first-order predicate* logic is not decidable, it cannot be a proper-logic part (logic subsystem) of a logically formalized system of proper *a-priori* knowledge. Consequently, the *first-order predicate* logic cannot be such a proper-logic basis of *universal* epistemology which is *common for both* the empirical knowledge subsystem and the *a-priory* knowledge subsystem of the knowledge system in general. Plenty of other intellectually respectable consistent logic systems (even more general, rich, and powerful than the *first-order predicate* logic) are either not complete, or *not decidable*, consequently, they are not acceptable candidates for the role of proper logic basis of/for a logically formalized *universal* epistemology *combining consistently* both the empiricism and the a-priori-ism in philosophy of knowledge. Taking the above-said into an account, it is quite reasonable to try exactly the propositional logic as a candidate for the role in question, because the propositional logic is consistent, complete, and *decidable,* and, consequently, *compatible* with the *a-priori*-ism in epistemology (while the *first-order predicate* logic and the *a-priori*-ism in epistemology are not compatible due to the theorem-schemes $(A\omega \supset (T\omega \leftrightarrow D\omega))$, $(A\omega \supset (P\omega \leftrightarrow D\omega))$. However, I agree that the *first-order predicate* logic is quite an adequate and in some concrete relations even the best proper-logic basis for logical formalization of many *empirical* knowledge systems.

6. *Formal proving the theorem-scheme* $(A\omega \supset ((C\omega \supset \neg Y\omega) \supset (C\omega \supset \neg PC\omega)))$ *in the theory Σ+2C, and discussing a logical consequence of combining this theorem-scheme with Bessonov's meta-theorem of logical independence of Gödel's second incompleteness meta-theorem from the first one*

"… the second theorem is independent of the first, …"
[Bessonov, 2022, pp. 5 and 8]

The scientific conference paper [Bessonov, 2022] has presented a proof of such a meta-theoretic statement, which means that the *second Gödel's incompleteness metatheorem does not follow logically from the first one*. I think that the meta-theoretic statement proved by Bessonov is nontrivial and deservs pondering over. In particular, concerning the mentioned meta-theoretic statement proved by Bessonov, in the present article, it is quite relevant to take into an account that, in Σ+2C, it is possible to construct a formal proof of the following theorem scheme to be called hearafter "Statement-III": $(A\omega \supset ((C\omega \supset \neg Y\omega) \supset (C\omega \supset \neg PC\omega)))$. The formal proof is the following queue 1)—13).

1) $A\omega$: assumption.
2) $(C\omega \supset \neg Y\omega)$: assumption.
3) $C\omega$: assumption.
4) $\neg Y\omega$: from 3 and 2 by *modus ponens.*
5) $\big(A\omega \supset (C\omega \leftrightarrow Y\omega)\big)$: the theorem scheme proved above (see the **Statement-I**).
6) $(C\omega \leftrightarrow Y\omega)$: from 5 and 1 by *modus ponens.*
7) $(C\omega \supset Y\omega)$: from 6 by the rule of elimination of $\leftrightarrow$.
8) $Y\omega$: from 7 and 3 by *modus ponens.*
9) $\neg PC\omega$: from 8 and 4 by the rule of introduction of $\neg$.
10) $A\omega, (C\omega \supset \neg Y\omega), C\omega \mid\!- \neg PC\omega$: due to the above succession 1—9.
11) $A\omega, (C\omega \supset \neg Y\omega) \mid\!- (C\omega \supset \neg PC\omega)$: from 10 by the rule of introduction of $\supset$.

12) Aω |– ((Cω ⊃ ¬Yω) ⊃ (Cω ⊃ ¬PCω))): from 11 by the introduction of ⊃.

13) |– (Aω ⊃ ((Cω ⊃ ¬Yω) ⊃ (Cω ⊃ ¬PCω))): from 12 by the introduction of ⊃.

The proof is finished. However, it could be continued in the given article in the following way with a view for utilizing and discussing the above-mentioned Bessonov's metatheoretic statement.

14) Aa_t |– ((Ca_t ⊃ ¬Ya_t) ⊃ (Ca_t ⊃ ¬PCa_t)): from 12 by substituting a_t for ω. (Here, "a_t" is a short name for the affirmation of arithmetic theory studied by Gödel.)

15) ¬((Ca_t ⊃ ¬Ya_t) ⊃ (Ca_t ⊃ ¬PCa_t)): the above-mentioned Bessonov's meta-theorem.

16) ¬Aa_t : from 14 and 15 by the rule of introduction of ¬.

Here we are. According to conjunction of the meta-theorem by Bessonov, and the given concrete *interpretation* of Σ+2C, the formal arithmetic theory in question is a representation of not *a-priori* but *empirical* knowledge. It is worth noting here that "a_t" is not a variable but a constant; in that concrete arithmetic interpretation which is discussed here, the constant "a_t" means perfectly fixed affirming the arithmetic theory studied by Gödel", and "Ca_t" means the statement that the arithmetic theory is consistent.

7. *Proving consistency of the formal theory Σ+2C*

For constructing the proof of consistency, we are to move from the *meta*-language of Σ+2C to the *object*-language of Σ+2C. Therefore, we are to move from the above-given schemes of axioms AX1—AX11 (and definition-scheme DF-1) to the following axioms AX1*—AX11* (and definition DF-1*), respectively.

Axiom AX-1*: Ap ⊃ (□q ⊃ q).

Axiom AX-2*: Ap ⊃ (□(p ⊃ q) ⊃ (□p ⊃ □q)).

Axiom AX-3*: Ap ↔ (Kp & (¬◊¬p & ¬◊Sp & □(q ↔ □q))).

Axiom AX-4*: Ep ↔ (Kp & (◊¬p ∨ ◊Sp ∨ ¬□(q ↔ □q))).

Axiom AX-5*: □p ⊃ ◊p.

Axiom AX-6*: (□q & □□q) ⊃ q.

Axiom AX-7*: (B_1^1x=+=C_1^1x) ↔ (G[B_1^1x] ↔ G[C_1^1x]).

Axiom AX-8*: (B_1^1x=+=g) ⊃ □G[B_1^1x].

Axiom AX-9*: (B_1^1x=+=b) ⊃ □W[B_1^1x].

Axiom AX-10*: (Gp ⊃ ¬Wp).

Axiom AX-11*: (Wp ⊃ ¬Gp).

Definition DF-1*: ◊p is a *name* of/for ¬□¬p, i.e. (◊p ↔ ¬□¬p) by definition.

Below in this paragrapg a precise definition is given of/for such a *function* $, which is an *interpretation* of the formal axiomatic theory Σ+2C (It is worth highlighting here that in this section of the article "t" denotes "true" and "f" denotes "false"). The interpretation-function $ is precisely defined below by the items 1—25.

1) For any formulae ω and π, and for any binary classical logic connective ⊕, it is true that $(ω ⊕ π) = ($ω ⊕ $π).

2) For any formula ω, it is true that $¬ω = ¬$ω.

3) $Ap = f.

4) $□q = f.

5) $\$q = t$.
6) $\$p = t$.
7) $\$\Box(p \supset q) = f$.
8) $\$\Box p = f$.
9) $\$Kp = t$.
10) $\$\Diamond\neg p = t$.
11) $\$\Diamond Sp = t$.
12) $\$\Box(q \leftrightarrow \Box q) = f$.
13) $\$Ep = t$.
14) $\$\Diamond p = t$.
15) $\$\Box\Box q = f$.
16) $\$(B_1{}^1x\text{=+=}C_1{}^1x) = t$.
17) $\$G[B_1{}^1x] = t$.
18) $\$G[C_1{}^1x] = t$.
19) $\$(B^1x\text{=+=}g) = f$.
20) $\$(B^1x\text{=+=}b) = f$.
21) $\$\Box G[B^1x] = f$.
22) $\$\Box W[B^1x] = f$.
23) $\$Gp = t$.
24) $\$Wp = f$.
25) $\$\Box\neg p = f$.

Under the *interpretation* \$ of the formal theory Σ+2C, the axioms AX1*—AX11* are true, the definition DF-1* is true, and the logic inference rules conserve truthfulness, hence, there is a model of/for Σ+2C, hence, the formal theory Σ+2C is consistent.

8. *Representing D. Hilbert's epistemic optimism by formal proving such a theorem which models that optimism in the theory Σ+2C to which the time-modality "it shall be in future that q" is added*

In the literature on philosophy of mathematics and on its history, when D. Hilbert's and Gödel's legacies are studied, very often their "rationalistic optimism" (or "epistemic optimism") is mentioned and discussed somehow [Ершов и Целищев, 2012], [Целищев, 2013], [Zach, 2019]. According to D. Hilbert's biography [Reid, 1996], his epistemic optimism has been expressed in his credo written on his gravestone in Göttingen, namely, "We must know. — We shall know", which credo has been formulated by Hilbert in response to the popular Latin maxim: "Ignoramus et ignorabimus" translated into English language by the sentence "We do not know. — We shall not know" [Reid, 1996, p 192].

Is it possible to represent (formulate precisely and prove deductively) the indicated Hilbert's credo within the theory Σ+2C? To answer this nontrivial question, first of all, let us think of possibility of a *relevant mutation* in the above-given definition of the set Δ of perfection modalities. According to the definition, Δ is finite, but, in principle, it may be extended by adding a new element (or a finite set of new elements) to it. Thus, in principle, the set Δ of perfection modalities is potentially infinite, i.e. open for any finite additions of qualitatively new perfection modalities. I believe that the time-modality "*in Future, it shall be so* that q" (or "it is Future that q", or "in Future, q") is a perfection modality. Let this time-modality be denoted by the sign "Б" (which is a letter belonging to the Russian language alphabet)[2] and added to Δ. If such adding is accepted then it is possible to construct the following proof scheme in Σ+2С+Б.

1) $A\alpha \leftrightarrow (K\alpha \,\&\, (\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta))$: axiom-scheme AX-3.
2) $A\alpha \supset (K\alpha \,\&\, (\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta))$: from 1 by elimination of $\leftrightarrow$.
3) $A\alpha$: assumption.
4) $(K\alpha \,\&\, (\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta))$: from 2 and 3 by *modus ponens*.
5) $\Box(\beta \leftrightarrow \Omega\beta)$: from 4 by elimination of &.
6) $(\beta \leftrightarrow \Omega\beta)$: from 3 and 5 by the rule of elimination of $\Box$.
7) $A\alpha \mid\!- (\beta \leftrightarrow \Omega\beta)$: by 1—6.
8) $A\alpha \mid\!- (\beta \leftrightarrow \Xi\beta)$: from 7 by substituting Ξ for Ω.
9) $A\alpha \mid\!- (\Xi\beta \leftrightarrow \beta)$: from 8 by commutativity of $\leftrightarrow$.
10) $A\alpha \mid\!- (\Xi\beta \leftrightarrow \Omega\beta)$: from 9 and 7 by transitivity of $\leftrightarrow$.
11) $A\alpha \mid\!- (\Xi\beta \leftrightarrow \Omega\beta)$: from 10 by the rule of introduction of $\supset$.
12) $\mid\!- \big(A\omega \supset (O\omega \leftrightarrow Б\omega)\big)$: from 11 by substituting: O for Ξ; and Б for Ω.
13) $\big(A\omega \supset (OK\omega \leftrightarrow БK\omega)\big)$: from 12 by substituting: $K\omega$ for ω.

Thus, constructing the proof scheme is finished. According to its philosophical interpretation, in Σ+2С+Б, the queue 1—12 is a model (representation) of justifying the optimism in general, and the queue 1—13 is a representation of vindicating D. Hilbert's epistemic optimism in particular. This is so because $(OK\omega \leftrightarrow Б\,K\omega)$ is translated into the natural language of humans by the sentence "we *must* know (that ...) — we *shall* know, *in future*, (that ...)". I think that this is exactly what Hilbert has believed in. The scientific novelty of this paragraph of the given article is precise defining the domain in which Hilbert's epistemic optimism is rationally justifiable. According to this paragraph, the domain, in which Hilbert's epistemic optimism is quite rational, is reduced to the small but not-empty realm of *pure a priori* knowledge exclusively. The strong psychological impression of oddity of the motto "We *must* know. — We *shall* know"[3] is quite an expectable outcome of *irrelevant* applying this motto to *empirical* knowledge.

9. *Conclusion*

I. Kant used to emphasize that proper mathematical knowledge is pure *a-priori*. This statement by Kant is especially highlighted not only in his "Critique of Pure Reason" [1994], but also in his "*Prolegomena*" [1996]. At first glance, Kantean philosophy of mathematics looks quite reasonable and truthlike, but, from conjunction of the axiomatic system Σ+2С and the famous

[2] I have decided to utilize the sign "Б" (the first letter of the Russian word "Будущее") here, as the sign "F" (the first letter of the English word "Future") is already occupied in Σ+2С.

[3] Extremist-minded positivists and empiricist-minded sceptics should evaluate this statement by Hilbert as either obviously *false* or *meaninless* one. I guess that they would prefer to agree with the Latin maxim: "We do not know. – We shall not know".

incompleteness theorems by Gödel it follows logically that Kant's general statement of a-priori-ness of mathematical knowledge is false. Moreover, it is formally provable in Σ+2C that ($A\omega \supset (\omega \leftrightarrow T\omega)$), consequently, those who do not agree with the equivalence ($\omega \leftrightarrow T\omega$), have to disagree with Kant's statement (modeled by $A\omega$) of a-priori-ness of mathematical knowledge system as well. Also, it is formally provable in Σ+2C that ($A\omega \supset (T\omega \leftrightarrow C\omega)$), consequently, if one negates ($T\omega \leftrightarrow C\omega$), then the one has to negate $A\omega$ as well. This means that the system of mathematical knowledge as a whole is not *a-priori*; as a whole it is an *empirical* one; only some small (but nevertheless very important) aspects of the whole mathematical knowledge system are *a-priori* ones. According to the above-said it is easy to see and formally to demonstrate that, in Σ+2C, Hilbert's ideal and program of philosophical grounding mathematics as a *self-sufficient* system *logically follow* from Kant's presumption that any proper mathematical statements and systems of knowledge are *a-priori* ones. If Kant's presumption was right, then, by means of Σ+2C, Hilbert's ideal and program would be well-grounded, convincingly explained, and vindicated totally. But the presumption by Kant is wrong, hence, significance of Hilbert's ideal and program is limited. However, notwithstanding the limitations it is quite adequate and works effectively within its own reduced but not-empty domain of applicability.

One could decide that those nontrivial conclusions about Kant's doctrine of a-priori knowledge which are obtained in the present article by virtue of the complicated multimodal formal axiology and epistemology theory Σ+2C, could be obtained more easiy by virtue of the so-called "natural logic" or "common sense". In first approximation, it seems that the hypothetical one is absolutely right. Nevertheless, I think that the one is right *not absolutely* but *only relatively*: only in some fixed concrete relations the notorious "common sense" attitude is quite reasonable. Very often simplicity is too expensive becauses it is achieved at the cost of significant loosing precision and rigor of discourse. Unfortunately, in the humanities confined in natural language and "natural logic" exclusively, there is no progress in debates of Kant's a-priori-ism; demonstrations of conclusions are not convincing. The present article is not mainly about Kant as a representative of the humanities, but mainly about D. Hilbert as a formalist-minded mathematician attempting at *rigorous* philosophical grounding mathematics as a self-sufficing system and exploiting Kant's a-priori-ism as a means of/for such attempting. In relation to Hilbert's program an analogous sceptic-ism can be developed. There are many respectable creative mathematicians, for example, H. Poincaré [2013], who prefer to use intuition, construction, and "natural logic" exclusively; they have a critical attitude to D. Hilbert's formalism and to B. Russell's logicism in philosophical foundations of mathematics and physics. Nevertheless, in my opinion, Hilbert's formalism is heuristically important: not always but sometimes it heads to ground-breaking nontrivial discoveries in some fields of scientific knowledge, for instance, in meta-mathematics and mathematical physics.

I guess that the above-presented (but not completely exhausted) trend of constructing and investigating multimodal formal axiomatic philosophy systems from Ξ, Σ, Θ, Σ+C to Σ+2C (and then further) is promising nontrivial discoveries in various fields of philosophical knowledge, namely, in proper philosophical formal ontology, formal epistemology, formal axiology, etc. The expected ground-breaking discoveries could have fruitful applications to philosophy of science, logic, ethics, aesthetics, jurisprudence, and even philosophical theology. Possibilities of some psychologically unexpected fruitful applications of the multimodal formal axiomatic philosophy systems to philosophical grounding physics or biology are not excluded here as well. I guess that substantial progress of the research submitted in the present article may be accomplished by investigating a possibility of developing the formal axiomatic epistemology-and-axiology theory Σ+2C further due to generating and selecting useful (heuristically fruitful) mutations in it. In relation to some possible application domains (for instance, to theoretical physics or theoretical biology), yet it is not quite clear whether the above-presented set of axiom-schemes of Σ+2C is sufficient for adequate mathematical modeling nontrivial philosophical problems arizing in these special domains.

At the present moment it is even not possible exactly to formulate some important aspects of the problem of completeness of Σ+2C. The *syntax* of Σ+2C is elaborated sufficiently and represented manifestly. However, the *content* aspect of semantics of Σ+2C, namely, the relationship between the standard interpretation of Σ+2C and yet *indefinite* (not quite restricted) application domain of Σ+2C, is to be investigated further. Probably, some *content* intuitions underlying the formal theory are not formulated manifestly as today they are too vague and not well-recognized. I feel that investigation of the *content* aspect of semantics of Σ+2C (its relationship to its external world) is to undergo significant development and elaboration in future. This hypothetical *content* analysis of still not completely defined subject-matter of the axiomatic epistemology-and-axiology theory Σ+2C can result in such significant changes in formal semantics and formal syntax of Σ+2C which changes transform Σ+2C into a qualitatively new formal theory. In particular, I guess that in future some qualitatively new nontrivial axiom-schemes are to be added to Σ+2C. In any way, the hypothetical heuristically useful mutations of Σ+2C and promising applications of results of its mutations are future vistas.

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